
DIRECTIONS To receive full credit, you must provide complete legible solutions to the following problems in the space provided. Transfer all your answers to the space provided on the test paper.

1. For what values of r does the function $y = e^{rx}$ satisfy the differential equation $2y'' + 5y' + 3y = 0$?
Ans _____

2. A population is modeled by the differential equation

$$\frac{dP}{dt} = 0.1P\left(1 - \frac{P}{1000}\right)$$

- a. For what values of P is the population increasing? Ans _____
- b. For what values of P is the population decreasing? Ans _____
- c. What are the equilibrium solutions? Ans _____

3. A function $y(t)$ satisfies the differential equation

$$y' = y^2 - 1$$

- a. What are the constant solutions of the equation? Ans _____
- b. For what values of y is y increasing? Ans _____
- c. For what values of y is y decreasing? Ans _____

3.a Show that every member of the family of functions $y = \frac{1 + ce^t}{1 - ce^t}$ is a solution of the differential equation $y' = \frac{1}{2}(y^2 - 1)$

3b. Find the solution of the differential equation that satisfies the initial condition $y(0)=10$.

Ans _____

4. Find the critical points and the phase portrait of the given autonomous 1st order differential equation, use a sign chart for both the first and second derivatives of y to draw the phase line and equilibrium solutions and sketch typical solution curves on a phase portrait per class notes.

$$\frac{dy}{dx} = y^2 - 7y + 10$$

