
Instructions: Write complete solutions to the following problems in the space provided. Be sure to supply all the necessary steps that lead to your answers

1. Find the mass and center of mass of the lamina that occupies the region D and has the given density function ρ .
D is the triangular region with vertices:
 $(0,0)$, $(2,1)$, $(0,3)$; $\rho(x,y) = 2(x+y)$
- Ans _____

2. Find the mass and center of mass of the lamina that occupies the region D and has the given density function ρ .
D is the triangular region enclosed by the lines:
 $x=0$, $y=x$, and $2x+y=6$; $\rho(x,y) = 2x^2$
- Ans _____

3. A lamina occupies the part of the disk in the first quadrant. Find its center of mass if the density at any point is proportional to its distance from the x-axis. $x^2 + y^2 \leq 4$
- Ans _____

4. Find the moments of inertia I_x, I_y, I_0 of a lamina in the shape of an isosceles right triangle with equal sides of length a if the density at any point is proportional to the square of the distance from the vertex opposite the hypotenuse. Assume that the coefficient of proportionality is k , and that the lamina lies in the region bounded by $x = 0, y = 0,$ and $y = a - x$
- Ans _____

5. The joint density function for a pair of random variables X and Y is given.
- Ans _____

$$f(x, y) = \begin{cases} Cx(1 + y) & \text{if } 0 \leq x \leq 1, 0 \leq y \leq 4 \\ 0 & \text{otherwise} \end{cases}$$

- a. Find the value of the constant C .
- b. Find $P(X \leq 1, Y \leq 1)$
- c. Find $P(X + Y \leq 1)$