SIMPLE HARMONIC MOTION

OBJECTIVE

To calculate the spring constant 'k' of a spring by using Hooke's Law and N2L and compare the results.

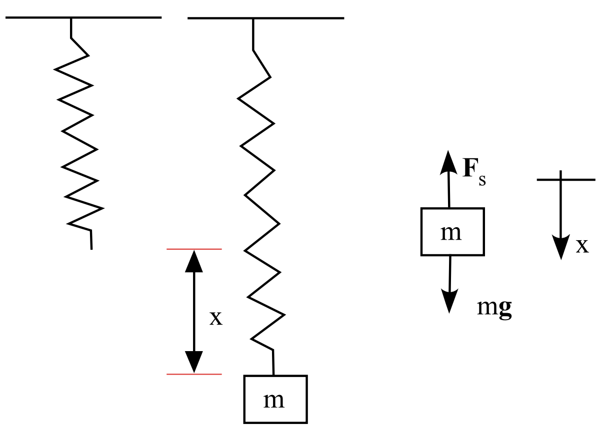
EQUIPMENT

1. 2-support rods and clamp
2. spring
3. masses and hanger
4. stopwatch
5. 2-m stick
6. Triple-Beam Balance

THEORY

I. *Using Hooke's Law*

Consider a spring suspended vertically in its equilibrium position. Suppose you add a mass 'm' to the end of the spring that displaces the spring an amount 'x' from equilibrium.



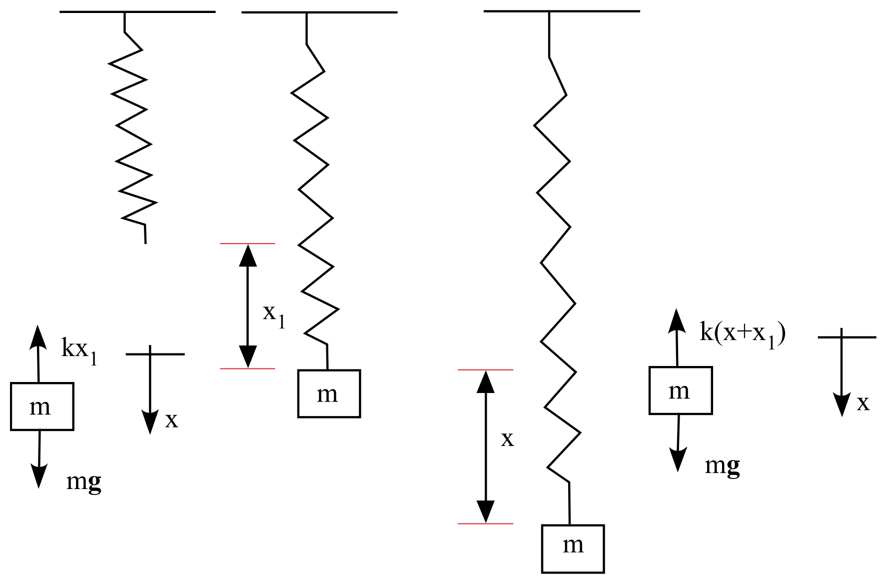
Applying N2L gives:

$$\sum F_x = mg - kx = 0$$
$$mg = kx$$

1. The graph of mg vs. x will give a straight line with the slope of the line equal to the spring constant 'k'.
2. We will take this value of 'k' to be the expected value.

II. *Using* N2L

Suppose you add a mass 'm' to the end of a suspended vertical spring. The mass displaces the spring an amount 'x_1' from equilibrium. In this position the mass is in equilibrium. Consider displacing the spring an amount 'x' from the new equilibrium position.



Applying N2L for the first displacement gives:

$$\sum F_x = mg - kx_1 = 0$$

$$mg = kx_1$$

Applying N2L for the second displacement gives:

$$\sum F_x = mg - k(x + x_1)$$

$$\sum F_x = mg - kx - kx_1$$

$$\sum F_x = kx_1 - kx - kx_1$$

$$\boxed{\sum F_x = -kx}$$ Net Force on Mass

$$\sum F_x = -kx = m\frac{d^2x}{dt^2}$$

$$\boxed{\frac{d^2x}{dt^2} + \left(\frac{k}{m}\right)x = 0}$$ Simple Harmonic Motion Equation

1. Confirm that the solution to this equation is given by:

$$\boxed{x(t) = A\cos(\omega t + \phi)}\text{ Solution to SHM Equation}$$

Where,

$x(t)$ = amplitude of oscillation (rad)
A = maximum amplitude of oscillations from equilibrium (rad)
$\omega = \sqrt{\frac{k}{m}}$ (angular frequency in units of rad/s) It is a measure of how fast the oscillations occur.
t = time (s)
φ = phase angle (rad) (determined by initial conditions)

2. The cosine and sin function repeat every period T. Thus:

$$\theta(t) = \theta(t+T)$$
$$\theta_m \cos(\omega t + \phi) = \theta_m \cos[\omega(t+T) + \phi)]$$
$$\theta_m \cos(\omega t + \phi) = \theta_m \cos[(\omega t + \phi) + \omega T)]$$

The sine and cosine repeat when their phase changes by 2π. Thus,

$$\omega T = 2\pi$$
$$T = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{\frac{k}{m}}} = 2\pi\sqrt{\frac{m}{k}}$$
$$T^2 = \left(\frac{1}{k}\right)4\pi^2 m$$

3. The graph of T^2 vs. $4\pi^2 m$ will give a straight line with the slope equal to 1/k.
4. We will take this 'k' value to be the experimental result and compare to the expected value.

PROCEDURE

1. Attach spring to horizontal rod and measure equilibrium position.
2. Attach 100g to end of spring and measure displacement 'x' from equilibrium.
3. Displace the mass slightly from equilibrium and release.
4. Measure the time for 10 oscillations and calculate the period. Repeat for a total of 3 runs.
5. Repeat steps (1) – (4) for the masses listed in the table below and fill in the rest of the data.
6. Make a graph of $\underline{mg\ vs.\ \text{x}}$ using EXCEL and from the equation of the best curve-fit determine the expected value of k.
7. Make a graph of $\underline{T_{ave}^2 vs.\ 4\pi^2 m}$ using EXCEL and from the equation of the best curve-fit determine the experimental value of k.
8. Compare both values of k.

DATA TABLE

m(gram)	mg	x	t1	T_1	t_2	T_2	t_3	T_3	T_{ave}	T_{ave}^2	$4\pi^2$ m
100											
150											
200											
250											
300											